Ranking-based Optimal Resource Allocation in Peer-to-Peer Networks

Yonghe Yan, Adel El-Atawy, Ehab Al-Shaer
School of Computer Science, Telecommunication and Information Systems
DePaul University
Chicago, IL 60604, USA

Abstract—**This paper presents a theoretic framework of optimal resource allocation and admission control for peer-to-peer networks. Peer's behavioral rankings are incorporated into the resource allocation and admission control to provide differentiated services and even to block peers with bad rankings. These peers may be free-riders or suspicious attackers. A peer improves her ranking by contributing resources to the P2P system or deteriorates her ranking by consuming services. Therefore, the ranking-based resource allocation provides necessary incentives for peers to contribute their resources to the P2P systems. We define a utility function which captures the best wish for the source peer to serve competing peers, who request services from the source peer. Although the utility function is convex, Harsanyi-type social welfare functions are devised to obtain a unique optimal resource allocation that achieves max-min fairness. The parameters used in our model can be derived from the nature of the services or chosen by the source peer. No private information is required to reveal from individual peers. This prevents selfish peers to play the system strategically and cheat the resource allocation mechanism for their own benefits. The resource allocation and admission control are fully distributed and linearly scalable.**

I. INTRODUCTION

Peer-to-Peer networks have become promising applications in the Internet. In the last few years P2P systems have increased dramatically in size, use, and underlying protocols [14], [17], [19], [22], [28], [31]. However, P2P systems suffer heavily from "free-rider" problem [1], [25]. Free-riders consume free services provided by other peers without contributing their resources to P2P systems. It leads an unfair society as well as degradation in overall performances of the networks. Therefore, healthy P2P systems should have mechanisms that provide appropriate incentives to encourage participant peers to contribute their resources to P2P systems.

In this paper, we consider a resource allocation mechanism, which allows source peers to provide competing peers differentiated services. Source peers provide services and allocate their resources to serve competing peers. These competing peers consume services and use the resources allocated by source peers. A ranking mechanism is incorporated in the resource allocations. The rankings are derived from peers' behavioral histories of providing services and consuming services. Therefore, a competing peer with a good history of serving other peers is provided a service with better quality, because the source peer will allocate more resources to serve the peer. This service differentiation provides incentives for peers to contribute their resources to P2P systems.

In the context of resource allocation problems, fairness is an important criterion for mechanisms that allocate shared resources. Borrowed from microeconomic theory, fairness is achieving by two ways: either maximizing the social welfare [12] or maximizing the minimum utility obtained, i.e., max-min fairness [23]. Although a special case of max-min fairness is well known in data communication literature [5], social welfare maximization is the prevalent notion for achieving fairness in bandwidth allocation problems [13], [24], [30]. These mechanisms are developed to provide differentiated services among heterogeneous users and/or to charge them prices, while achieving social welfare maximization. The social welfare functions are some forms of aggregate utility functions of individual users. The individual utility functions are specified with parameters, which represent the service classes (or types) of users. The parameters that specify the service classes should be considered private information of individual users. Recent studies [15], [16], [27] show that a user may misrepresent his/her utility function strategically and deviate from the optimal social allocation. Therefore, these mechanisms cannot achieve optimal allocation if a user does not truthfully reveal his/her utility. Therefore, the resource allocation mechanism we proposed does not require a user to reveal his/her private information.

The resource allocation we proposed utilizes publicly observable and verifiable information to achieve optimal resource allocation. The ranking incorporated in the resource allocation is derived from peer's behavioral history, which is a peer's history of providing services and consuming services. Since a service session always involves two peers, these behavioral histories can be observed and verified by both peers who involved in the service session. Therefore it does not require a user to reveal his/her private information unilaterally. Moreover, the parameters used for individual utility functions are devised by the source peer. Since it is the source peer's intention to achieve optimal resource allocation, the source peer does not have any incentives to misrepresent the utility functions. The utility functions are to capture the best wish for the source peer to serve individual competing peers, and the utility functions are convex. Since the utility functions are not concave, social welfare maximization cannot be the fairness criterion in our resource allocation. Fortunately, max-min fairness can be achieved easily by a unique optimal allocation in our model.

Since behavioral histories are incorporated in the proposed resource allocation mechanism, the mechanism even has the potential to alleviate security attacks, such as flooding attacks, on source peers. The admission control can even block such malicious peers to be served by source peers.

The paper is organized as follows: Section II discuss some related work. The resource allocation problem and the model used are presented in Section III. Utility functions and Harsanyi-type social welfare functions are introduced in this section as well. In Section IV, we discuss the ranking framework that is incorporated in the resource allocation. Section V considers the admission control for the source peer to admit peers with good rankings. We illustrate our model by evaluating sample networks in Section VI. Finally, Section VII draws conclusions and future work.

II. Related Work

There has been considerable research on overlay routing networks such as CAN [19], Chord [22], Pastry [14], and Tapestry [31]. These systems assume that all participant peers will follow the well-defined protocols and do not consider any incentives for participant peers. P2P networks based these protocols will eventually suffer from free-rider problem. Therefore, incentives (e.g., reputations or prices) for participant peers are introduced into P2P networks [4], [7], [18], [20], [21]. Blanc et al [2] studied an incentive mechanism to encourage peers to participate P2P routing. Sanghavi and Hajek [26] proposed an incentive mechanism for peers to consume public goods (public goods are non-exclusionary resources, which does not involve resource allocation among multiple concurrent users).

When source peers provide differentiated services and/or charge prices, competing peers may select a subset of source peers to minimize the cost or to obtain the best services from a subset of source peers. Ader et al considered such optimal peer selection problem in [4]. Source peer selection problems were also investigated in [6] and [20], especially, Habib and Chuang [9] proposed a peer selection mechanism that rewards contributors with flexibility and choice in source peer selection.

Bandwidth allocations with differentiation and incentive mechanism were investigated in [24]. The mechanism is developed by maximizing the aggregate utility. As mentioned before, the mechanism may suffer because untruthful users may misrepresent their utility strategically.

Although there has been considerable research in this area, our unique contributions in this paper are: 1) a mechanism of optimal resource allocation that achieves max-min fairness with convex utility functions, 2) service differentiations are derived from publicly observable and verifiable information so that it can prevent untruthful peers to misrepresent their utilities and even to alleviate malicious attacks on P2P systems. And the resource allocation mechanism proposed in the paper is fully distributed and linearly scalable by its nature.

III. Resource Allocation Problem

When a source peer allocates resources to serve other peers, we assume the source peer makes decision independently so that the resource allocation mechanism is linearly scalable. Without loss of generality, we assume that one peer in our model is the source peer who provides the services. Other peers are competing peers, who consume the services provided by the source peer. Let set $\mathbf{N}$ be the set of competing peers, and $N = |\mathbf{N}|$ is the number of these peers. The competing peers compete to share the source peer's resources.

While a single source peer and multiple competing peers are considered in our model, the model does not preclude a peer to consume services from multiple source peers. Since each source peer serves competing peers independently, it is competing peer's decision which subset of source peers the competing peer should request services from (see [4] and [9] for optimal peer selection). Therefore, each source peer only needs to satisfy the competing peers that are requesting the source peer to serve them. A salient property of this model is that it is fully distributed and linearly scalable since each source peer makes resource allocation decision independently.

The source peer utilizes behavioral histories of individual competing peers to provide differentiated services. Individual peer's behavioral history, i.e., providing services and/or consuming services, can be observed by both peers of a service session and submitted to a trusted third party. The trusted third party could be such a system as that has been proposed by [3]. The trusted third party also provides behavioral histories to the source peer when the peer wants to make decision on resource allocations. Even if a P2P system does not have access to such a trusted third party, a peer could keep local behavioral histories of other peers when those peers requested services from the peer or provided services to the peer. These local histories could allow a source peer to recognize friends from strangers. Therefore, the friendship could be measured by ranking the behavioral histories. Behavioral history systems or reputation systems are not in the scope of this paper, interested readers may refer to [3] or [18].

Competing peers are rated by the source peer based on the peers' behavioral histories. The ranking $p_i > 0$ of peer i is given by

$$p_i = f(c_i, s_i)$$

where $f(c_i, s_i)$ is the ranking function, which is increasing in c_i and decreasing in s_i. The c_i and s_i are peer i's histories of consuming services and providing services respectively. Therefore, a good friend is rated with a small number and total stranger is rated with a large number. Note that a suspicious malicious peer could also be rated with a large number and hence the ranking has the potential to be used for alleviating malicious attacks, such as flooding attacks. A friend is a peer who has an observed history to serve the source peer or other peers. Therefore, the higher p_i is, the less peer i has contributed to the P2P system.

We assume that the source peer should allocate R_i unit of resources to serve competing peer i with good quality. When fewer resources are allocated to the competing peer, the quality of service is worse. However, the minimum resources to sustain the service or to provide the service with the minimum acceptable quality is r_i. Therefore, the source peer should allocate x_i unit of resources to peer i and $R_i \geq x_i \geq r_i \geq 0$. Since the source peer could derive R_i and r_i from the nature of the services (e.g., audio, video or file transfer), they may not be considered private information. And this prevents a user to misrepresent his/her resource requirements strategically and to play the allocation mechanism solely for his/her own benefits.

Assume that the resource capacity of the source peer is $C > 0$. Since the total resources required by all competing peers cannot exceed the recourse capacity, the minimum resource requirements implies that the total minimum resources cannot exceed the source peer's resource capacity, that is, the minimum resources $\mathbf{r} = (r_1, \cdots, r_N)^T$ has to satisfy the source peer's capacity constraint, i.e., $\sum_{i=1}^{N} r_i \leq C$. Without this assumption, it is not feasible for the source peer to find an allocation that meets the minimum resource requirements of all the competing peers. In the rest of the paper, we implicitly consider feasible minimum resource requirements only. We will revisit this issue later in admission control.

Since R_i is the resources needed to serve peer i with good quality of service, the best way for the source peer to serve the peer is to allocate R_i unit of resources to the peer. Therefore we call R_i the target resources. When an allocation deviates from the target resources, the quality of service deteriorates. Therefore, a competing peer has an aversion to any resource allocations that deviate from the target resources. Thus, to satisfy the competing peer, the source peer should allocate resource x_i that maximizes the following utility function over the interval $[r_i, R_i]$

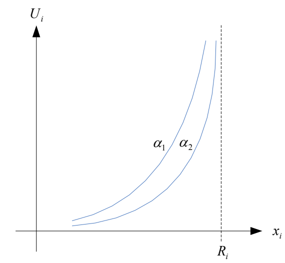

Figure 1. Utility function, $\alpha_1 < \alpha_2$

$$U_i(x_i) = \frac{1}{(\alpha - 1)(R_i - x_i)^{\alpha - 1}}, \quad i = 1, \cdots, N \tag{1}$$

where the parameter $\alpha > 1$ represents the sensitivity to the deviation from target resources. As depicted in Fig. 1, the utility is more sensitive to deviations when α is large and vice versa. This utility function reflects that the source peer wishes to serve the peer with resource R_i since $U_i(x_i) \to \infty$ when $x_i \to R_i$. When x_i deviates from R_i, the utility decreases sharply because the peer's quality of service deteriorates.

The utility function simply says that the source peer wishes to serve the competing peer with the target resources. A natural question is when the total target resources of all competing peers exceed the resource capacity of the source peer, how should the source peer allocate resources among the competing peers. In other words, when the source peer is congested, competing peers cannot be allocated the target resources and the resource capacity should be fairly shared by the competing peers. Because of the capacity constraint, allocating more resources to one competing peer implies allocating fewer resources to other peers. To solve this conflicting problem and to allocate resources fairly among competing peers, we are interested in constructing a mechanism to achieve max-min fairness.

To construct the resource allocation mechanism, we first associate a competing peer with a Harsanyi-type social welfare function [10]

$$U(x_i, \mathbf{x}_{-i}) = \sum_{k=1}^{N} w_k U_k(x_k) \tag{2}$$

where $\mathbf{x}_{-i} = (x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_N)^T$, $U_k(x_k)$ is the utility function of competing peer k defined in (1) and $w_k = p_k (R_k - r_k)^{\alpha}$. The parameter w_k is the weight of the peer's utility in the social welfare function. It takes into account the ranking and the resource requirements. The parameter is so set that the marginal cost of resource capacity expansion becomes the threshold ranking for the source peer to serve a peer. We will revisit it later when we address ranking framework.

A Harsanyi-type social welfare function is associated with an individual user. The social observer of the system, i.e., the source peer, concerns the well-being of an allocation. Hence, to achieve a fair allocation, the source peer allows a competing peer to maximize the minimum social welfare the peer could obtain, that is,

$$\max_{x_i} \min_{\mathbf{x}_{-i}} U(x_i, \mathbf{x}_{-i}) \text{ , } i = 1, \cdots, N \text{ .} \tag{3}$$

subject to

$$\sum_{i=1}^{N} x_i \le C \text{ .}$$

And the solution to problem (3) is an allocation that achieves the max-min fairness.

When the total target resources required by all competing peers do not exceed the resource capacity (i.e., $\sum_{i=1}^{N} R_i \le C$), problem (3) can be solved trivially. The source peer simply allocates the target resources to every competing peer. Therefore, when the source peer is not congested, everyone is provided the service with good quality. This could help attracting more users to join the P2P systems. However, we are more interested to solve problem (3) when the source peer is congested (i.e., $\sum_{i=1}^{N} R_i > C$) and then the source peer provides differentiated services.

When the source peer is congested, an allocation should satisfy the capacity constraint $\sum_{i=1}^{N} x_i = C$. Hence we have $x_i = C - \sum_{k \ne i} x_k$. Substituting it into (3), problem (3) becomes one minimization problem $\min_{\mathbf{x}} U(x_i, \mathbf{x}_{-i})$ and

$$\min U(\mathbf{x}) = \min \sum_{i=1}^{N} w_i U_i(x_i) \tag{4}$$

subject to

$$\sum_{i=1}^{N} x_i = C \tag{5}$$

$$\mathbf{x} \le \mathbf{R} \tag{6}$$

$$\mathbf{x} \ge \mathbf{r} \tag{7}$$

The constraint (5) expresses the source peer's capacity constraint. Resource requirements of competing peers are represented by (6) and (7).

Remark: The minimization in (4) seems counterintuitive. It is actually the dramatic reflection of Arrow's impossibility theorem [12], that is, social welfare functions could not be rational in the sense of individual preferences. The minimization is achieved with the fullest use of resource capacity. Because the minimum social welfare is maximized, it assures the well-being of the source peer and the allocation is fair. ■

Assume that minimum resource constraint (7) is satisfied. Hence problem (3) is defined on the set that is nonempty, convex, and compact. And $U(\mathbf{x})$ is a convex function since its Hessian matrix $\nabla^2 U(\mathbf{x})$ is positive definite. This implies that a unique solution to problem (4) exists when the minimum resource constraint (7) holds. And the minimum resource requirements could be enforced by an admission control later in section V.

Let $\mathcal{L}(\mathbf{x}, \mu, \boldsymbol{\lambda})$ denote the Lagrangian where $\mu > 0$ and $\lambda_i \ge 0$, $i = 1, \cdots, N$, are the Lagrange multipliers associated with constraint (5) and (6), respectively. Then

$$\mathcal{L}(\mathbf{x}, \mu, \boldsymbol{\lambda}) = U(\mathbf{x}) - \mu\left(\sum_{k=1}^{N} x_k - C\right) - \boldsymbol{\lambda}^T(\mathbf{x} - \mathbf{R})$$

The first-order Kuhn-Tucker conditions [11] are

$$\frac{w_i}{(R_i - \dot{x}_i)^{\alpha}} - \mu - \lambda_i = 0 \text{ , } i = 1, \cdots, N \text{ ,}$$

and

$$\lambda_i(\dot{x}_i - R_i) = 0 \text{ , } \lambda_i \ge 0 \text{ , } i = 1, \cdots, N \text{ ,}$$

$$\sum_{i=1}^{N} \dot{x}_i = C \text{ , } \mu > 0 \text{ ,}$$

$$\dot{x}_i \ge r_i \text{ , } i = 1, \cdots, N \text{ .}$$

When the source peer is congested, we see that the constraint $\dot{\mathbf{x}} \le \mathbf{R}$ is inactive and hence $\lambda_i = 0$ for all $i = 1, \cdots, N$. Thus, we have the solution to problem (3)

$$\dot{x}_i = R_i - \left(\frac{w_i}{\mu}\right)^{1/\alpha} \text{ , } i = 1, \cdots, N \text{ ,} \tag{8}$$

$$\sum_{i=1}^{N} \dot{x}_i = C \text{ , } \dot{x}_i \ge r_i \text{ , } i = 1, \cdots, N \tag{9}$$

This solution is the optimal resource allocation that achieves max-min fairness. It is also one of Nash equilibriums [8] of problem (3).

Proposition: A resource allocation is a Nash equilibrium if constraint (5) holds.

Proof: Suppose that $\mathbf{x}^* = (x_i^*, \mathbf{x}_{-i}^*)$ is a resource allocation that satisfies constraint (5). Assume that peer i is allocated resource x_i such that $x_i \ne x_i^*$. The resource allocation $(x_i, \mathbf{x}_{-i}^*)$ must be feasible so that $x_i + \sum_{k \ne i} x_k^* \le C$. Combining these relations, we have $x_i \le x_i^*$. Because $U(x_i, \mathbf{x}_{-i})$ is a strictly increasing function with respect to x_i, we obtain

$$U(x_i, \mathbf{x}_{-i}^*) \le U(x_i^*, \mathbf{x}_{-i}^*) \text{ , } i = 1, \cdots, N \text{ .}$$

Therefore, we have shown that $\mathbf{x}^*$ is a Nash equilibrium of problem (3). ■

It is clear that solution (8) satisfies constraint (5) and therefore it is a Nash equilibrium.

Remark: We have mentioned before that there are two fairness criteria for allocation problems: social welfare maximization and max-min fairness. The resource allocation (8) achieves max-min fairness. Given the social welfare function (2), we will show that social welfare maximization cannot result in a "fair" allocation. That is, the source peer may allow each competing peer to maximize his/her social welfare function

$$\max_{\mathbf{x}} U(x_i, \mathbf{x}_{-i}) = \max_{\mathbf{x}} U(\mathbf{x})$$

Note that $\partial U(\mathbf{x})/\partial x_i > 0$ and $\partial^2 U(\mathbf{x})/\partial^2 x_i > 0$ on the interval $[0, R_i)$. Hence, while maximizing $U(\mathbf{x})$, x_i may keep increasing until it reaches R_i. Therefore, the allocation for peer *i* is one of values from the set

$$\{0, R_i, C - \sum_{k \neq i} \eta_k R_k \mid \eta_k \in \{0,1\}\} \quad (10)$$

The third value in (10) is the remaining resource capacity of the source peer when some other competing peers reach their target resources. Although the source peer is congested, some competing peers are still allocated resources that reach their target resources. In this case, these competing peers are fully satisfied and they will not be affected by the congestion at the source peer. At the same time, some other competing peers are allocated zero or the remaining resource capacity. Therefore, this is not considered a "fair" resource allocation when there is congestion at the source peer. Moreover, ranking mechanism cannot be incorporated in these resources allocations since the weight w_k cannot effect allocations. ■

IV. Ranking Framework

When we define the utility function (1) and social welfare function (2) in the previous section, two parameters, w_i and α, are used in these definitions. These two parameters are closely related to ranking mechanism that is incorporated in our model. We are going to investigate the ranking framework in this section.

We have shown that when the source peer is congested, the Lagrange multiplier $\mu > 0$ is a positive number in the solution to problem (3). From microeconomic theory, we know that the Lagrange multiplier in (8) can be interpreted as the marginal cost for resource capacity expansion at the source peer [29]. Therefore we take the marginal cost to be the threshold ranking $\overline{p}$ that a competing peer needs for the source peer to serve the peer, thus

$$\overline{p} = \mu \,. \quad (11)$$

Since peer *i* needs at least resource r_i, combining (8) and (11), and let $\dot{x}_i \geq r_i$, yields

$$\dot{x}_i = R_i - \left(\frac{w_i}{\overline{p}}\right)^{1/\alpha} \geq r_i \,.$$

Then, we obtain $w_i \leq \overline{p}(R_i - r_i)^{\alpha}$. Note that we set $w_i = p_i (R_i - r_i)^{\alpha}$ in social welfare function (2) and therefore we have

$$p_i \leq \overline{p}\,,\; i = 1, \cdots, N \,. \quad (12)$$

It is clear that as long as the ranking is not greater than the threshold ranking, a competing peer is allocated resources that satisfy the minimum resource requirement of the competing peer. And the resources allocated to the competing peer is a function of the ranking p_i,

$$\dot{x}_i = R_i - (R_i - r_i)\left(\frac{p_i}{\overline{p}}\right)^{1/\alpha}$$

Since the threshold ranking $\overline{p}$ is a function of all rankings, say, $\overline{p} = f(p_1, \cdots, p_N)$, if a competing peer is among a large number of competing peers and the impact of the competing peer's ranking on $\overline{p}$ is negligible, we see that the resources allocated to peer *i* is a decreasing function with respect to p_i. Therefore the rankings allow the source peer to provide differentiated services based on their rankings.

Assume that the source peer is congested and thus, from (8) and (9), we have

$$\dot{x}_i = R_i - (R_i - r_i)\left(\frac{p_i}{\mu}\right)^{1/\alpha}$$

$$\sum_{i=1}^{N} \dot{x}_i = C$$

Solving these equations, we obtain

$$\dot{x}_i = (1 - \beta_i) R_i + \beta_i\, r_i \,,\; i = 1, \cdots, N \,, \quad (13)$$

where

$$\beta_i = \frac{\sum_{k=1}^{N} R_k - C}{\sum_{k=1}^{N} (R_k - r_k)(p_k / p_i)^{1/\alpha}}$$

It shows that the ranking ratios rather than the rankings alone have effect on the resource allocation. It implies that there are competitions among competing peers because it is the ranking ratio changes that can cause allocation change. Moreover, the parameter α controls the sensitivity of the changes.

When $\alpha \to \infty$, we have $\beta_\infty = \lim_{\alpha \to \infty} \beta_i$, and

$$\beta_\infty = \frac{\sum_{k=1}^{N} R_k - C}{\sum_{k=1}^{N} (R_k - r_k)} \,. \quad (14)$$

Therefore, ranking ratios do not have any effect on the resource allocation when $\alpha \to \infty$. Note that for any feasible resource requirements we have

$$\sum_{k=1}^{N} R_k \geq C \geq \sum_{k=1}^{N} r_k \,,$$

hence we see that $0 \leq \beta_\infty \leq 1$. Consequently, the resource allocation given by (13) always satisfies the resource requirements of a competing peer (i.e., $R_i \geq x_i \geq r_i$), and the resource allocation is only determined by the source peer's resource capacity and resource requirements of competing peers. This leads to a non-differentiated resource allocation algorithm at the source peer.

When α is a small number, the resource allocation is more sensitive to the rankings. Therefore, when the source peer is suffering from free-riders or suspicious attackers, the source peer should choose a small α. Otherwise, a larger α may be chosen by the source peer.

When peer i's ranking is worse than the threshold ranking (i.e., $p_i > \overline{p}$), the parameter β_i in (13) is going to be greater than one and the source peer cannot allocate the competing peer resources that satisfies his/her minimum resource requirement. Therefore, the source peer should not admit such competing peer with this bad ranking and an admission control mechanism should be integrated into the solution to problem (3).

V. Admission Control

The discussions in section IV show that a competing peer with bad ranking may result in a resource allocation that will not satisfy the minimum resource constraints. When we enforce the minimum resource constraint, we obtain the admission conditions.

From (8), we have

$$\begin{aligned} \dot{x}_i &= R_i - (R_i - r_i)\left(\frac{p_i}{\mu}\right)^{1/\alpha} \\ &= (1-\beta_i)R_i + \beta_i r_i \end{aligned}$$

where $\beta_i = (p_i/\mu)^{1/\alpha}$. Thus, $\dot{x}_i$ is guaranteed to be between target resources R_i and minimum resources r_i when $0 \le \beta_i \le 1$.

Propositio. Admission Conditions: The source peer is able to allocate competing peers resources that satisfy their resource requirements if the admission conditions holds:

$$0 \le \beta_i \le 1,\ i = 1, \cdots, N. \tag{15}$$

where

$$\beta_i = \left(\frac{p_i}{\overline{p}}\right)^{1/\alpha},\quad i = 1, \cdots, N.$$

and the optimal allocation $\dot{\mathbf{x}}$ is given by:

$$\dot{x}_i = (1-\beta_i)R_i + \beta_i r_i,\ i = 1, \cdots, N, \tag{16}$$

$$\textstyle\sum_{i=1}^{N} \dot{x}_i = C. \tag{17}$$

Proof: Equation (16) and (17) give a resource allocation that is the solution to problem (4) without the minimum resource constraint. Combining the admission conditions (15) and (16), the resource allocation satisfies the resource requirement, $\mathbf{r} \le \dot{\mathbf{x}} \le \mathbf{R}$, for all competing peers. Therefore, the resource allocation satisfies all the constraints of problem (4) and the source peer can admit these competing peers. ■

The admission control guarantees that admitted competing peers are allocated resources that satisfy the resource requirements even if the source peer is congested and a peer with a bad ranking will not be admitted by source peer.

For a given set of competing peers with feasible minimum resource requirements (i.e., $\sum_{i=1}^{N} r_i \le C$), rankings are the decision variables of the admission conditions (15). We have shown that if the ranking of a competing peer is greater than the threshold ranking, the admission conditions (15) will not be satisfied and the competing peer will not be admitted by the source peer. Therefore, a competing peer has to manage to contribute to the P2P system and to get a better ranking for him/her to be served. Moreover, the threshold ranking is a function of rankings of all competing peers. Therefore, there are competitions for the competing peers to satisfy the admission conditions.

When the source peer is not congested, we see that β_i is not well-defined. However, only constraint (6) is active when there is no congestion. Therefore, the solution to problem (3) is a resource allocation that every competing peer is allocated the target resources. And no admission controls are needed. Rankings do not play any role for admission decision and resource allocation either. Therefore, the source peer just provides services with good quality to all competing peers when the source peer is not congested.

To explain the admission conditions further, we consider a simple network with two competing peers. Suppose that the source peer's capacity is C, peer 1 and peer 2 have the ranking p_1 and p_2, and require target resource R_1 and R_2, minimum resource r_1 and r_2, respectively. We should also assume that $R_1 + R_2 \ge C \ge r_1 + r_2$ so that it is feasible for these two competing peers to compete for sharing the source peer's resources. Assume that when the two competing peers attempt to share the source peer, congestion occurs at the source peer. Thus, the admission conditions are

$$0 \le \beta_i \le 1, \tag{18}$$

$$\dot{x}_i = (1-\beta_i)R_i + \beta_i r_i, \tag{19}$$

$$\dot{x}_1 + \dot{x}_2 = C, \tag{20}$$

$$\beta_i = \left(\frac{p_i}{\overline{p}}\right)^{1/\alpha}, \tag{21}$$

where $i = 1, 2$. Combining (19), (20), and (21), we obtain

$$\beta_1 = \frac{R_1 + R_2 - C}{(R_1 - r_1)p_1^{1/\alpha} + (R_2 - r_2)p_2^{1/\alpha}} p_1^{1/\alpha},$$

$$\beta_2 = \frac{R_1 + R_2 - C}{(R_1 - r_1)p_1^{1/\alpha} + (R_2 - r_2)p_2^{1/\alpha}} p_2^{1/\alpha}$$

Substituting β_1 and β_2 into (18), and rearranging the terms, we see that these two competing peers are able to share the source peer if the following conditions are satisfied

$$\left(\frac{p_1}{p_2}\right)^{1/\alpha} \ge \frac{R_1 + r_2 - C}{R_1 - r_1} \tag{22}$$

$$\left(\frac{p_2}{p_1}\right)^{1/\alpha} \ge \frac{r_1 + R_2 - C}{R_2 - r_2}. \tag{23}$$

Hence, when the ranking ratio satisfies these conditions, condition (18) is satisfied, and the source peer can have a resource allocation that satisfies the competing peers' resource requirements (i.e., $R_i \geq \dot{x}_i \geq r_i$). When peer 1's ranking is much worse than peer 2, condition (23) may be violated. On the other hand, condition (22) may be violated when peer 2's ranking is much worse than peer 1. Therefore, the ranking of one competing peer is related to the ranking of the other competing peer when the source peer allocates them resources. When condition (22) reaches equality, we have $\beta_2 = 1$, $\dot{x}_2 = r_2$ and $\dot{x}_1 = C - r_2$. And peer 2 has worse ranking than peer 1, but it is still within the threshold for the source peer to serve peer 2. If peer 2's ranking is further worse than that, the source peer will not serve the peer because condition (22) is violated. Conversely, when condition (23) reaches equality, we see that $\beta_1 = 1$, $\dot{x}_1 = r_1$ and $\dot{x}_2 = C - r_1$. When neither condition (22) nor (23) takes the equality, both competing peers are allocated resources between target resources and minimum resources.

We have seen that when the ranking of a competing peer is bad enough, the peer will be blocked by the source peer from services. Therefore, the admission control does not only guarantee the admitted competing peers to be served with satisfactory quality but also blocks peers with bad rankings, such peers could be free-riders or suspicious attackers. On other hand, when the source peer is not congested, the source peer will serve all the peers with good quality of service even if the peers are free-riders. This helps the P2P system to attract new users to join the P2P system. It is necessary for the healthy growth of the P2P system.

VI. EVALUATION

In this section, we show examples that illustrate the discussions in previous sections

Suppose that there are four competing peers that request services from the source peer, the minimum resources of all competing peers are zero, and the source peer's capacity is 1. Table I and Table II show the resource allocations with two different sets of rankings. When α = 50, we see that the rankings have little effect on the resource allocations since the resource allocations in Table I and Table II are almost the same. From (16) and (17), we obtain $\dot{x}_i = R_i C \big/ \sum_{k=1}^{4} R_k = 0.4 R_i$ for $i = 1, \cdots, 4$ when $\alpha \to \infty$. That is, each competing peer is allocated resources that are 40% of the target resources and the resource allocation is (0.1, 0.2, 0.3, 0.4). Therefore, when α is large enough, the resources allocated to individual competing peer should be approximately 40% of target resources. When α = 1.01 or α = 2, the resource allocations in Table I and Table II are different because of the different rankings. In Table I, peer 1 is allocated 0.051 (α = 1.01) and 0.076 (α = 2) unit of resources that are less than 40% of the target resources since peer 1's ranking is relatively worse than other peers. However, peer 1 is allocated 0.141 (α = 1.01) and 0.123 (α = 2) unit of resources in Table II where peer 1's ranking is better than other peers. Thus, a competing peer is allocated resources that are more than 40% of the target resources when the ranking of the peer is relatively smaller than rankings of other peers. These tables also show that the allocations are more sensitive to the rankings when α is smaller.

TABLE I. RESOURCE ALLOCATION WITH RANKING SET 1

Peer	R	Ranking	Resource Allocation		
			$\alpha = 1.01$	$\alpha = 2$	$\alpha = 50$
1	0.25	2.00	0.051	0.076	0.099
2	0.50	1.75	0.150	0.175	0.199
3	0.75	1.50	0.300	0.298	0.300
4	1.00	1.25	0.499	0.450	0.402

TABLE II. RESOURCE ALLOCATION WITH RANKING SET 2

Peer	R	Ranking	Resource Allocation		
			$\alpha = 1.01$	$\alpha = 2$	$\alpha = 50$
1	0.25	1.25	0.142	0.123	0.101
2	0.50	1.50	0.242	0.222	0.201
3	0.75	1.75	0.300	0.299	0.300
4	1.00	2.00	0.315	0.357	0.398

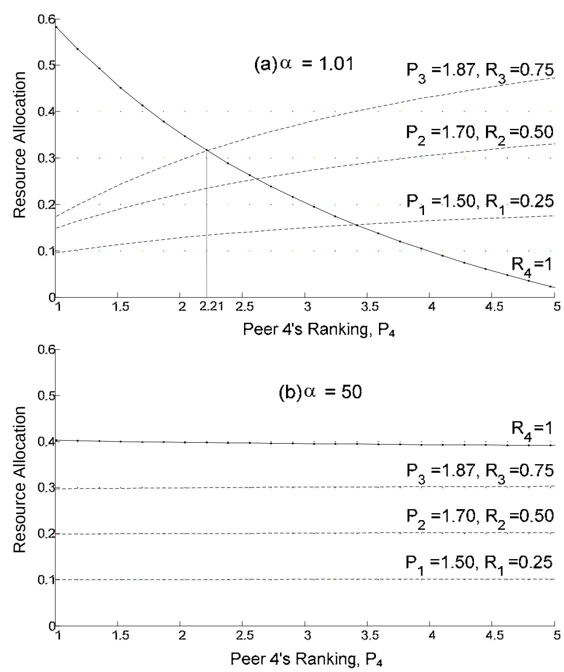

Figure 2. Resources and Ranking of peer 4.

Fig. 2 shows that resource allocation is a decreasing function of the ranking of a competing peer. The x-axis represents the ranking of peer 4 and the y-axis represents resource allocation. In Fig. 2.(a) and Fig. 2.(b), α = 1.01 and α = 50, respectively. While the ranking of peer 4 increases, the resources allocated to peer 4 decreases in Fig. 2.(a). Although peer 4's target resources are greater than peer 3's, the resources allocated to peer 4 will be less than that of peer 3 when $p_4 > 2.21$. Note that the rankings of peer 1, 2, and 3 in Fig. 2 are fixed values. The resources of these competing peers (dotted lines) increase in Fig. 2.(a) because the resources of peer 4 decreases. Resource allocation in Fig. 2.(b) do not change much while the ranking of peer 4 changes. Therefore, we see that the ranking of a competing peer has a strong effect on the resource allocation when α is small. On other hand, rankings have little effect on resource allocation when α is large enough. With appropriately chosen α, the source peer can balance the effect of rankings on resource allocations.

We have shown that a good ranking may help a peer to consume a service with better quality. When the source peer provides services to other peers, the source peer is also improving his/her ranking. And it is highly likely in a P2P system that the source peer will request services from other peers. Therefore, there is an incentive for a source peer to provide good services to other peers, so that this peer can earn a better ranking and be treated as a friend of other peers when he/she requests services from these peers. Thus, there is an incentive for the source peer to contribute more available resources when congestion occurs at the source peer.

In the next experiment, we will show the effect of contributing more resources by the source peer. We assume that there are four competing peers and the resource requirements are **R** = (1, 2, 3, 4) and **r** = (0.25, 0.5, 0.75, 1). Suppose that all the peers have the same ranking. Fig. 3 shows a linear increase when the source peer contributes more resources (i.e., increases the resource capacity). Therefore, every competing peer perceives a better service when the source peer increases the resource capacity.

Suppose that resource requirements of four competing peers are **R** = (1, 4, 1, 4) and **r** = (0.25, 1, 0.25, 1), the first two peers have the same rankings and the other two peers' rankings are also the same but different from the first two peers. The rankings of peer 1 and peer 2 are better than the rankings of peer 3 and peer 4. That is, the first two peers are friends and the other two peers are strangers of the P2P system. The resources allocated to a friend as shown in Fig. 4 is higher than that to a stranger, although they have the same resource requirements. For strangers, we see that they suffer more if their resource requirements are higher. Therefore, there are strong incentives for a peer to improve his/her ranking when the peer is going to request resource-consuming services from other peers.

Now we assume that four competing peers require the same resource $R_i = 1$ and $r_i = 0.25$, for $i = 1, 2, 3, 4$. However, their rankings are different, that is, **p** = (1, 2, 3, 4). As we see in Fig. 5, when the resource capacity is just enough to satisfy the minimum requirements of these four peers, every peer is allocated the minimum resources. As more resource capacity becomes available, friends are given higher priority in distributing the extra available resources. However, this cannot go beyond the target resources. When the resource capacity approaches total amount of all target resources, the differences between friends and strangers diminish. It consists with that the source peer should satisfy all competing peers with the target resources when the source peer is not congested.

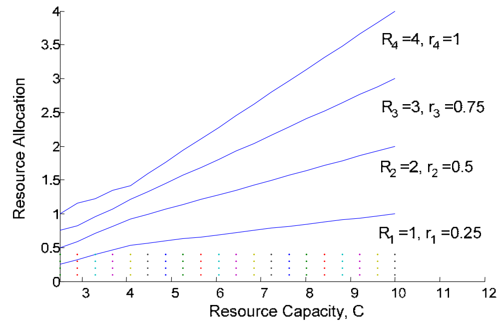

Figure 3. Resource allocation and resource capacity

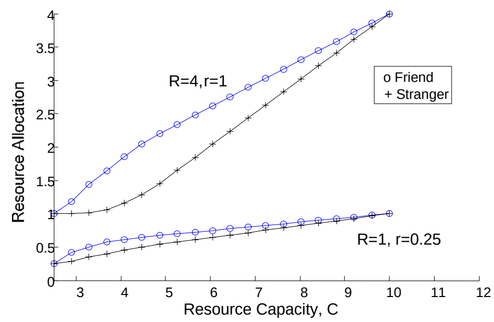

Figure 4. Resource allocation to friends and strangers

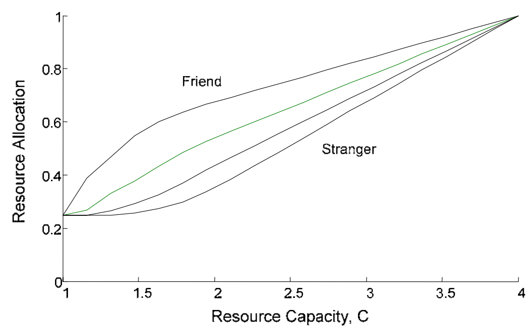

Figure 5. Resource allocation with different rankings.

VII. Conclusion and Future Work

In this paper, we have presented an optimal resource allocation and admission control for P2P systems. The optimal resource allocation achieves max-min fairness, which maximizes the minimum social welfare obtained. Rankings are incorporated in the resource allocation for the source peer to provide differentiated services. A peer improves his/her ranking by contributing resources to the P2P systems and deteriorates his/her ranking by consuming services. A peer with bad ranking is allocated fewer resources than peers with better rankings so that the peer perceives a worse quality of service. This provides incentives for a peer to improve his/her ranking by contributing resources to the P2P systems. The admission control can block peers with bad rankings. These peers may be free-riders or suspicious attackers. Moreover, the resource allocation and admission control are fully distributed and linearly scalable by the nature of the model.

A utility function was defined to capture the best wish for the source peer to serve the competing peers. The parameters used in the utility function can be derived from the nature of services so that no private information is required to reveal from competing peers. Therefore, competing peers cannot play the system strategically for their own benefits and cheat the resource allocation mechanism to get better services than that the peers deserve.

The evaluations demonstrate that the model is well tunable using parameter α and the rankings work well to have effect on resource allocations. In this paper, only stationary competing peers are considered. Dynamics of competing peers and resource scheduling should be considered in future work.

References

[1] E. Adar and B. Huberman. Free riding on Gnutella. Technical Report, Xerox PARC, August, 2000.

[2] Alberto Blanc, Yi-Kai Liu, and Amin Vahdat, Designing Incentives for Peer-to-Peer Routing, Proceedings of the IEEE Infocom Conference, Miami, FL, March 2005.

[3] M. Allman, E. Blanton and V. Paxson, An Architecture for Developing Behavioral History, Proc. Steps to Reducing Unwanted Traffic on the Internet (SRUTI) 2005.

[4] Micah Adler, Rakesh Kumar, Keith Ross, Dan Rubenstein, Torsten Suel, and David D. Yao. Optimal Peer Selection for P2P Downloading and Streaming. *Proceedings of IEEE INFOCOM*, March, 2004.

[5] D. Bertsekas and R. Gallager, Data Networks. Englewood Cliffs, NJ: Prentice Hall, 1987.

[6] Byung-Gon Chun, Rodrigo Fonseca, Ion Stoica, and John Kubiatowicz. Characterizing Selfishly Constructed Overlay Routing Networks. *Proceedings of the 23rd IEEE International Conference on Computer Communications*, 2004.

[7] Daniel Figueiredo, Jonathan Shapiro, and Don Towsley. Incentives for Cooperation in Anonymity Systems. *Proceedings of the 13TH IEEE International Conference on Network Protocols*, November, 2005.

[8] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, Cambridge, Massachusetts, 1996.

[9] A. Habib and J. Chuang, Incentive Mechanism for Peer-to-Peer Media Streaming, 12th IEEE International Workshop on Quality of Service (IWQoS'04), June 2004.

[10] J.C., Harsanyi. Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility. *The Journal of Political Economy*, Vol. 63-4, pp.309-321, 1955.

[11] Michael Hoy, John Livernois, Chris McKenna, Ray Rees, Thanasis Stengos, *Mathematics for Economics*, 2nd Edition, MIT Press, 2001.

[12] Kreps, D. M. A Course in Microeconomic Theory. Princeton University Press, Princeton, New Jersey, 1990.

[13] F. P. Kelly, A. Maulloo, and D. Tan, Rate control in communication networks: shadow prices, proportional fairness and stability, Journal of the Operational Research Society, vol. 49, pp. 237–252, 1998.

[14] A. Rowstron and P. Druschel. Pastry: Scalable, decentralized object location, and routing for large-scale peer-to-peer systems. *Lecture Notes in Computer Science*, vol. 2218, 2001.

[15] Johari, R. and Tsitsiklis, J.N. A scalable network resource allocation mechanism with bounded efficiency loss. To appear in IEEE Journal on Selected Areas in Communications, 2006.

[16] Johari, R., and Tsitsiklis, J. Efficiency loss in a network resource allocation game. Mathematics of Operations Research 29, 3, p407-435, 2004.

[17] M. Ripeanu. *Peer-to-peer architecture case study: Gnutella network.* Technical Report, Computer Science Dept., University of Chicago, 2001.

[18] Paul Resnick, Ko Kuwabara, Richard Zeckhauser, and Eric Friedman. Reputation systems. *Communications of the ACM*, 43(12):45–48, 2000.

[19] Sylvia Ratnasamy, Paul Francis, Mark Handley, Richard Karp, and Scott Shenker. A scalable content-addressable network. *Proceedings of ACM SIGCOMM 2001*, August, 2001.

[20] S. Ratnasamy, M. Handley, R. Karp, and S. Shenker. Topologically-aware overlay construction and server selection. *Proceedings of IEEE INFOCOM'02*, New York, NY, June 2002.

[21] N. Ben Salem and M. Jakobsson L. Buttyan, J.P. Hubaux. A charging and rewarding scheme for packet forwarding. *Proc. of MobiHoc*, June 2003.

[22] I. Stoica and R. Morris and D. Karger and M. F. Kaashoek and H. Balakrishnan. Chord: A scalable peer-to-peer lookup service for internet applications. *Proceedings of ACM SIGCOMM,* 2001.

[23] Rawls, J. A Theory of Justice. Belknap Press, 1971.

[24] Richard T. B. Ma, Sam C. M. Lee, John C. S. Lui, David K. Y. Yau, A Game Theoretic Approach to Provide Incentive and Service Differentiation in P2P Networks Sigmetrics/Performance 2004.

[25] S. Saroiu, P. K. Gummadi, and S. D. Gribble. A measurement study of peer-to-peer files sharing systems. *Proceedings of Multimedia Computing and Networking,* January, 2002.

[26] S. Sanghavi and B. Hajek, A New Mechanism for the Free-rider Problem, the Third Workshop on Economics of Peer-to-Peer Systems, 2005.

[27] S. Sanghavi and B. Hajek, Optimal Allocation of a Divisible Good to Strategic Buyers, 2005.

[28] S. Saroiu, K. P. Gummadi, and S. D. Gribble, Measuring and analyzing the characteristics of Napster and Gnutella hosts, Multimedia System, vol. 9, no. 2, pp. 170–184, 2003.

[29] H.R. Varian. *Microeconomic Analysis*. Third edition, Norton: New York, 1992.

[30] Haïkel Yaïche, Ravi R. Mazumdar, and Catherine Rosenberg, A Game Theoretic Framework for Bandwidth Allocation and Pricing in Broadband Networks, IEEE/ACM Transactions on Networking, Vol. 8, No. 5, October 2000.

[31] B. Y. Zhao, L. Huang, J. Stribling, S. C. Rhea, A. D. Joseph, and J. Kubiatowicz. Tapestry: A resilient global-scale overlay for service deployment. *IEEE Journal on Selected Areas in Communications*. January 2004.